Study of momentum diffusion with the effect of adiabatic focusing

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Goal

- The influence of the adiabatic focusing effect on the momentum diffusion.

Background

- Momentum diffusion of the charged energetic particles is an important mechanism of the transport process in astrophysics, physics of the fusion devices, and laboratory plasmas.
- According to observations the background magnetic field of the magnetized plasma is usually nonuniform.
- Schlickeiser & Shalchi (2008) and Litvinenko & Schlickeiser and so on found the focused field can leads to an new first-order acceleration mechanism.
- By combining the momentum convective term and the momentum derivative term, for some special cases Armstrong et al. (2012) derived the formulas of mean momentum change rate.

Method

- The combination of the perturbation and iteration method (Wang et al. 2017; Wang & Qin 2018, 2019, 2020)

- The scale analysis method used by Gombosi et al. (1993).

The equation of the isotropic distribution function for the homogeneous field

$$\frac{\partial F}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\kappa^c_{pp}\frac{\partial F}{\partial p}\right] + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\kappa^c_{3p1}\frac{\partial}{\partial p}\left(\kappa^c_{3p2}\frac{\partial F}{\partial p}\right)\right]$$
$$+\frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\kappa^c_{4p1}\frac{\partial}{\partial p}\left[\kappa^c_{4p2}\frac{\partial}{\partial p}\left(\kappa^c_{4p3}\frac{\partial F}{\partial p}\right)\right]\right\} + \cdots \quad (1)$$

The equation of the isotropic distribution function with the adiabatic focusing effect

$$
\frac{\partial F}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\kappa_p^f F\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\kappa_{pp}^c\frac{\partial F}{\partial p}\right] + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\mathcal{M}(\epsilon,\xi)\frac{\partial F}{\partial p}\right)
$$
$$
+\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\kappa_{3p1}^f\frac{\partial}{\partial p}\left(\kappa_{3p2}^f\frac{\partial F}{\partial p}\right)\right] + \frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\kappa_{4p1}^f\frac{\partial}{\partial p}\left[\kappa_{4p2}^f\frac{\partial}{\partial p}\left(\kappa_{4p3}^f\frac{\partial F}{\partial p}\right)\right]\right\}
$$
$$
+\cdots, \tag{2}
$$

with

$$
\epsilon = \frac{v_A}{v}, \qquad \xi = \frac{\lambda}{L}.
$$

In this research, we set $\epsilon \sim \xi \sim \eta$.

The new momentum diffusion term

Comparing Equations (1) and (2), we can find that the focused field contributes to

- a momentum streaming term

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\kappa_p^f F\right),$$

which was explored by Schlickeiser & Shalchi (2008) and Litvinenko & Schlickeiser (2011),

- an additional second-order momentum derivative term

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\mathcal{M}(\epsilon,\xi)\frac{\partial F}{\partial p}\right),$$

which is new.

The momentum diffusion coefficient

The momentum diffusion coefficient exact up to the fourth-order in η

$$\mathcal{M}_4(\epsilon,\xi) = D_1\left[\frac{2}{9}\epsilon^3 H_C\left(7-20H_C^2\right)\xi + \epsilon^2\left(\frac{188}{135}H_C^2 - \frac{2}{45}\right)\xi^2\right]p^2$$

- For the divergent field, i.e., $\xi > 0$

$$\mathcal{M}_4(\epsilon,\xi) \sim D_1\left(-\frac{2}{45} + \frac{14}{9}H_C + \frac{188}{135}H_C^2 - \frac{40}{9}H_C^3\right)p^2\eta^4$$

- For the convergent field, i.e., $\xi < 0$

$$\mathcal{M}_4(\epsilon,\xi) \sim D_1\left[-\frac{2}{45} - \frac{14}{9}H_C + \frac{188}{135}H_C^2 + \frac{40}{9}H_C^3\right]p^2\eta^4$$

The sign of $\mathcal{M}_4(\epsilon,\xi)$ is determined by H_C and focusing parameter ξ.

The mean momentum change rate

The mean momentum change rate with time contributed from the new second-order momentum derivative term as

$$\left(\frac{d\langle p\rangle}{dt}\right)_{22} \approx \left\langle \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\mathcal{M}_4(\epsilon,\xi)\right)\right\rangle = 4D_1\alpha$$

with

$$\alpha = \frac{2}{9}\epsilon^3 H_C\left(7-20H_C^2\right)\xi + \epsilon^2\left(\frac{188}{135}H_C^2 - \frac{2}{45}\right)\xi^2$$

The mean momentum change rate

$$\langle p\rangle_{22} \sim \langle p\rangle_0 e^{4D_1\alpha\eta^4 t}$$

The focused field provides an additional momentum loss or gain process

Conclusions

- The second-order momentum derivative term caused by the focused field is different from the one by the constant field, so it is identified that the focused field provides an additional momentum loss or gain process.

- This physical process should occur as long as the background magnetic field is converging or diverging.

Future Work

- The more general description for the momentum transport contributed from the focused field.

- The cross-derivative term over time and momentum.

- The fractional-order momentum derivative equation.


Thank you!