

# Ultra-high energy cosmic rays in harmonic space

*Stefano Camera*

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UNIVERSITY of the  
WESTERN CAPE



Physics

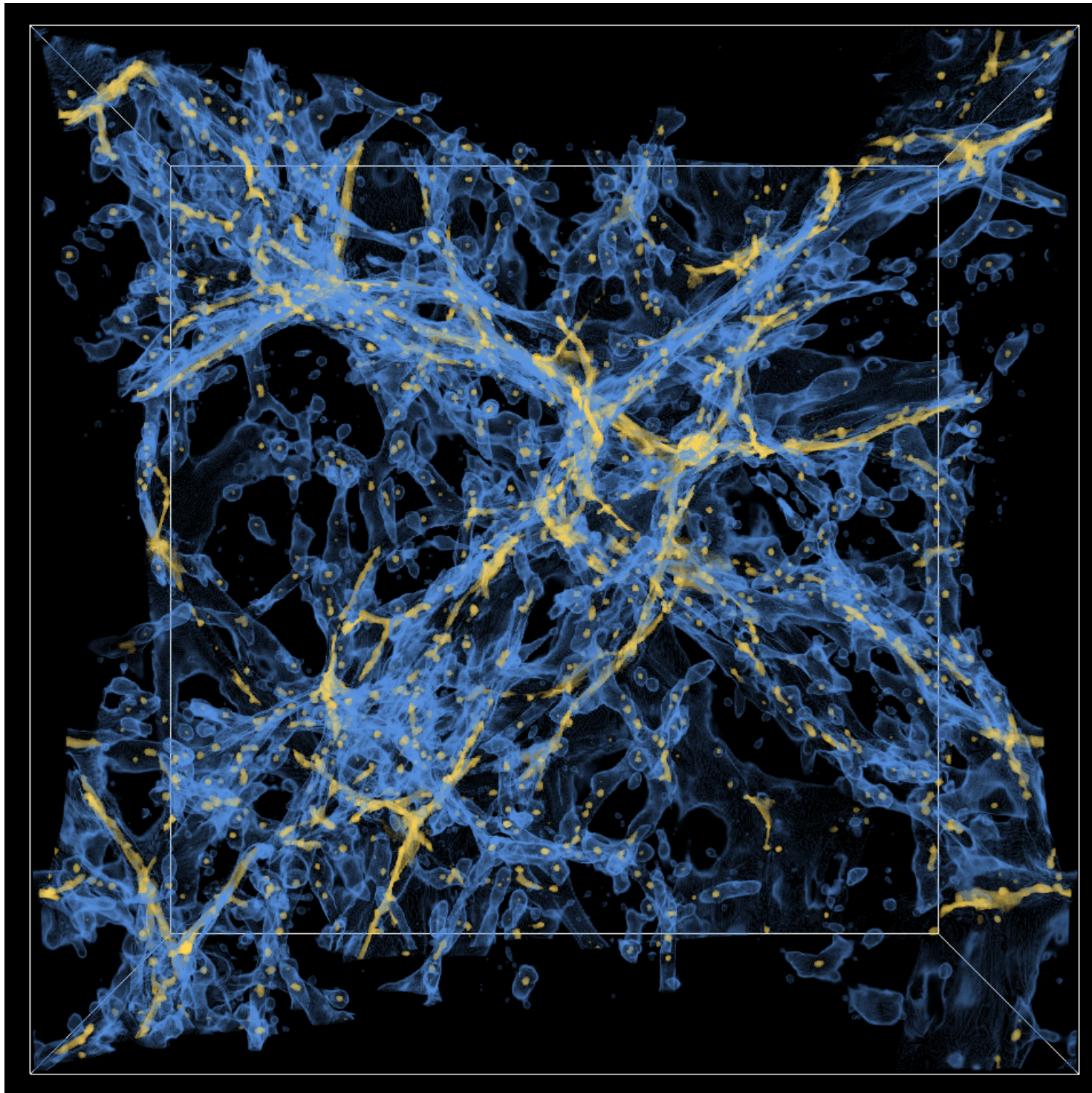


# Razionale



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TAURINENSIS

[Lukic et al.; Image: Casey Stark]



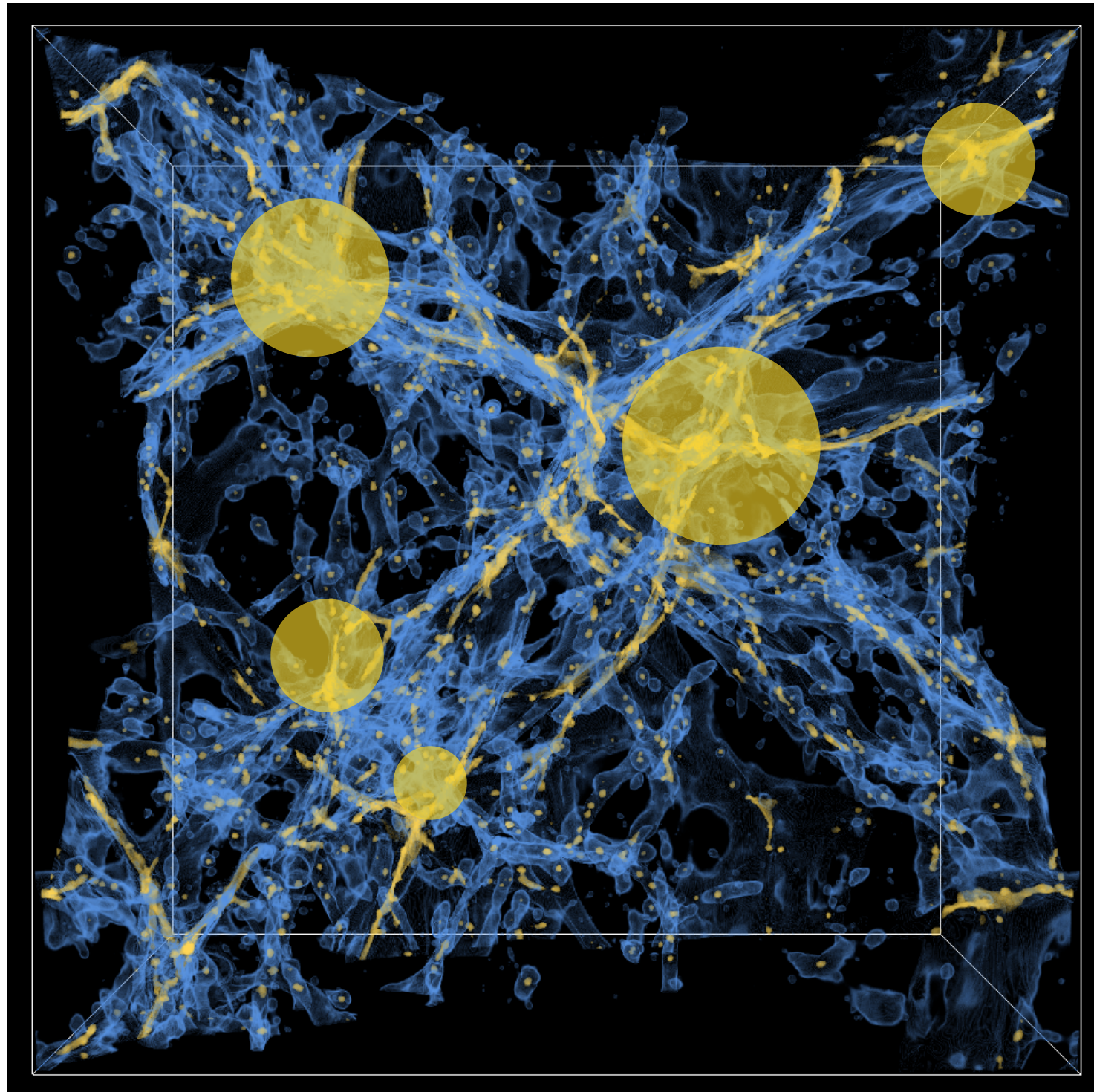


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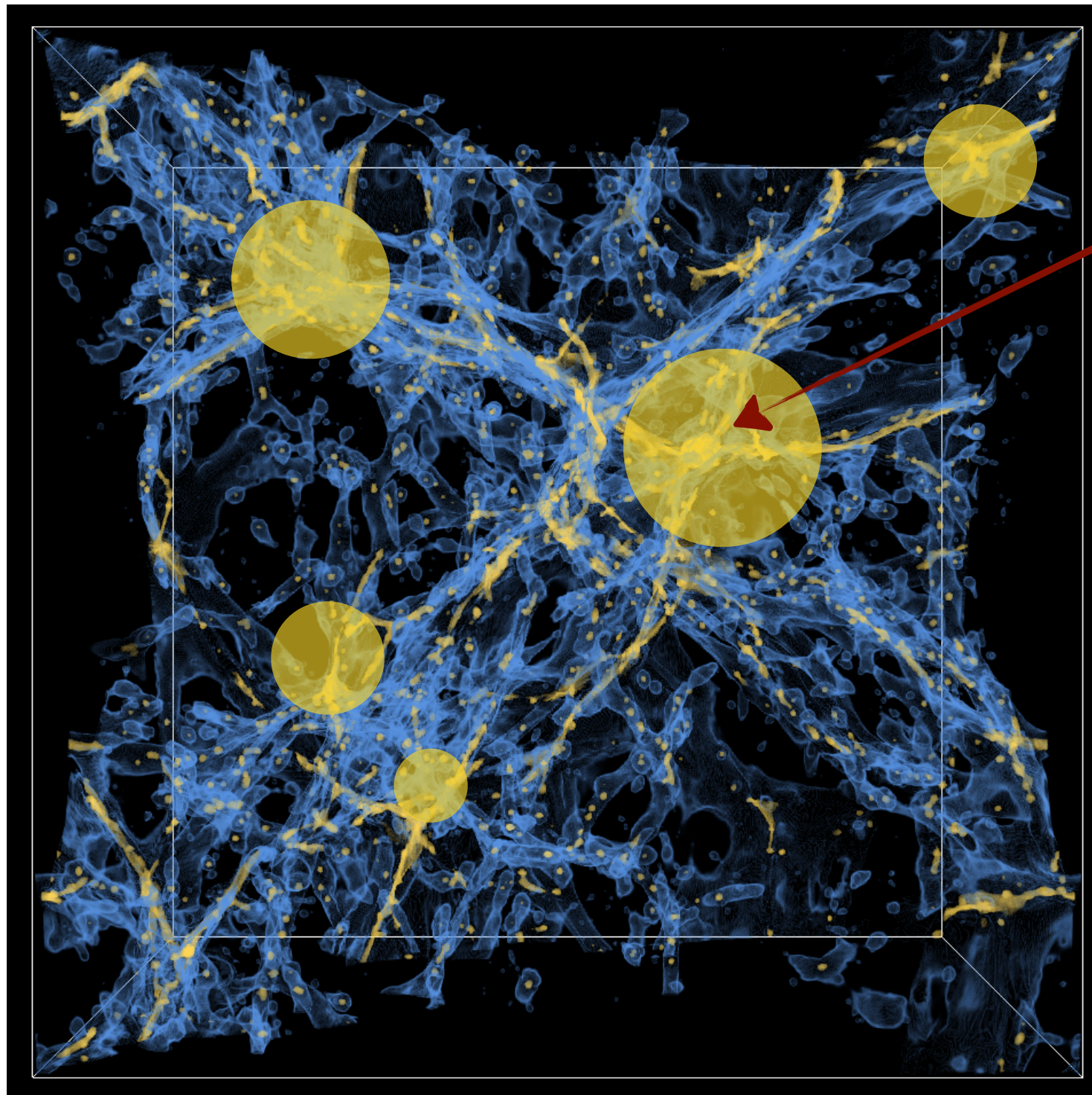


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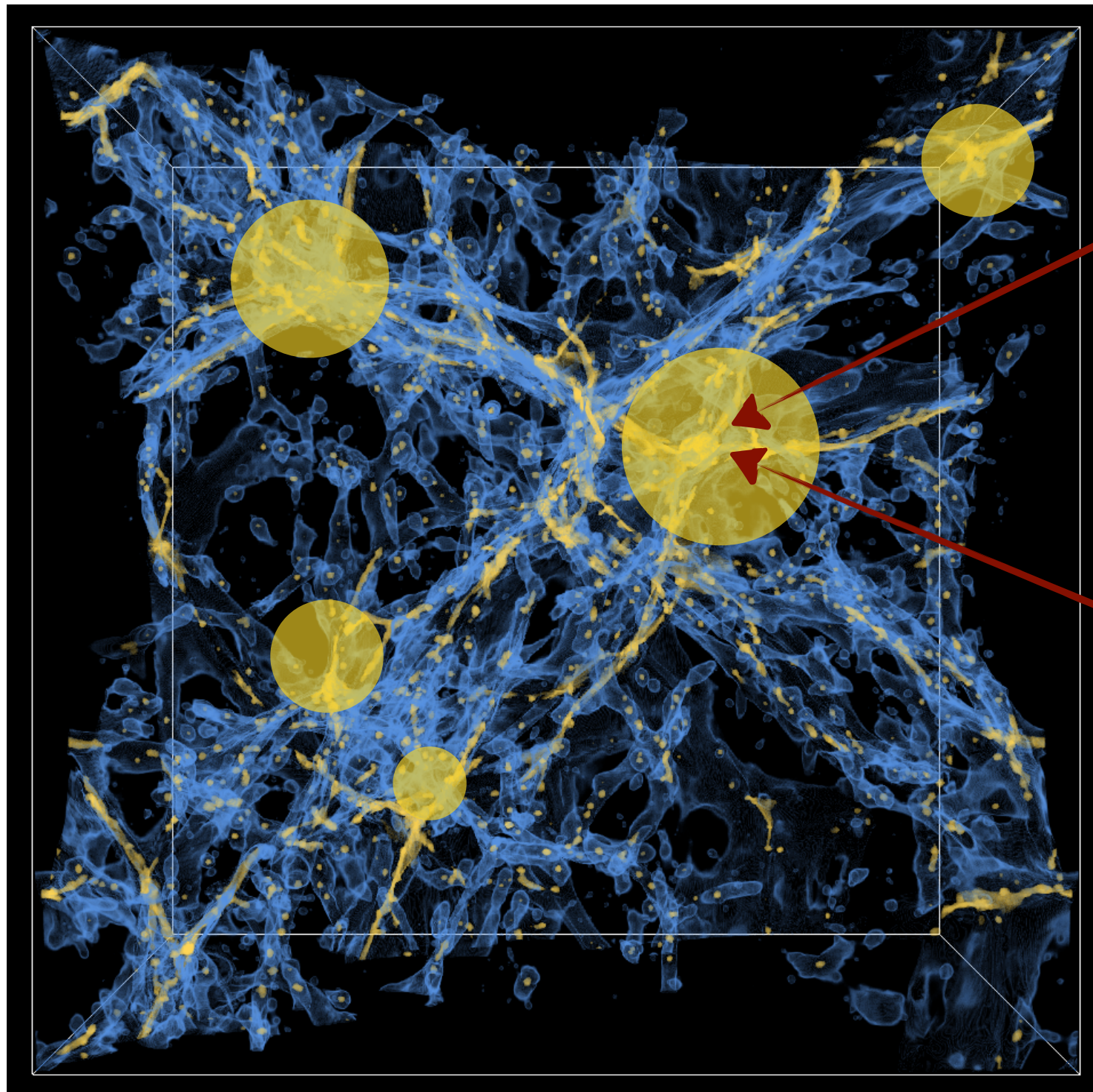


Galaxies,  
galaxy clusters,  
gravitational lensing



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[Lukic et al.; Image: Casey Stark]



Galaxies,  
galaxy clusters,  
gravitational lensing

Cosmic rays from  
astrophysical sources  
hosted within the  
dark matter halo

# Correlations 1.01



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- Cosmological perturbations  
[temperature anisotropies, density fluctuations...]

$$f(t, \boldsymbol{x})$$

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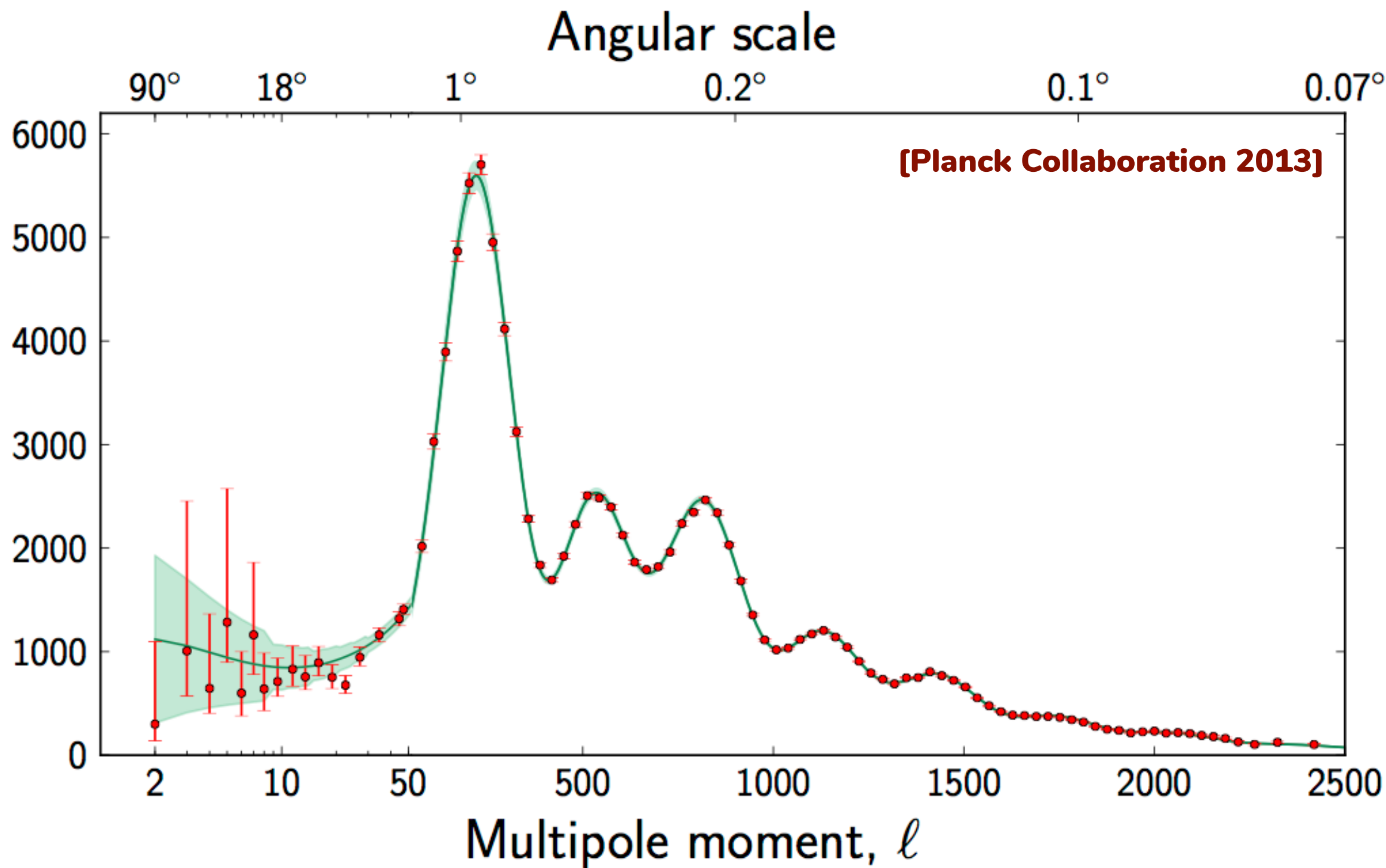
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- E.g.: CMB temperature anisotropies  $f(t, \mathbf{x}) \rightarrow T(t_0, \hat{\mathbf{n}})$



# Correlations 1.01



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# Cross-correlations

- Cosmological perturbations  $f(t, \boldsymbol{x}), g(t, \boldsymbol{x})$   
[temperature anisotropies, density fluctuations...]
- Correlation function  $\xi^{fg}(t, |\boldsymbol{x} - \boldsymbol{y}|) = \langle f(t, \boldsymbol{x})g(t, \boldsymbol{y}) \rangle$
- Fourier-space power spectrum  
 $\langle \hat{f}_{\boldsymbol{k}}(t)\hat{g}_{\boldsymbol{k}'}^*(t) \rangle = (2\pi)^3 \delta_{\text{D}}(\boldsymbol{k} - \boldsymbol{k}')P^{fg}(k, t)$
- Harmonic-space power spectrum  
 $\langle \tilde{f}_{\ell m}(z)\tilde{g}_{\ell' m'}^*(z') \rangle = \delta_{\ell\ell'}^{\text{K}}\delta_{mm'}^{\text{K}}C_{\ell}^{fg}(z, z')$
- E.g.: CMB-galaxy cross-correlation (measurement of **ISW**)





# UHECR anisotropies

- Fluctuations in detected UHECRs in a given direction on the sky and above a certain energy cut

$$\Delta_{\text{CR}}(\hat{\boldsymbol{r}}, E_{\text{cut}}) = \frac{\text{Flux}(\hat{\boldsymbol{r}}, E_{\text{cut}})}{\text{MeanFlux}(E_{\text{cut}})} - 1$$



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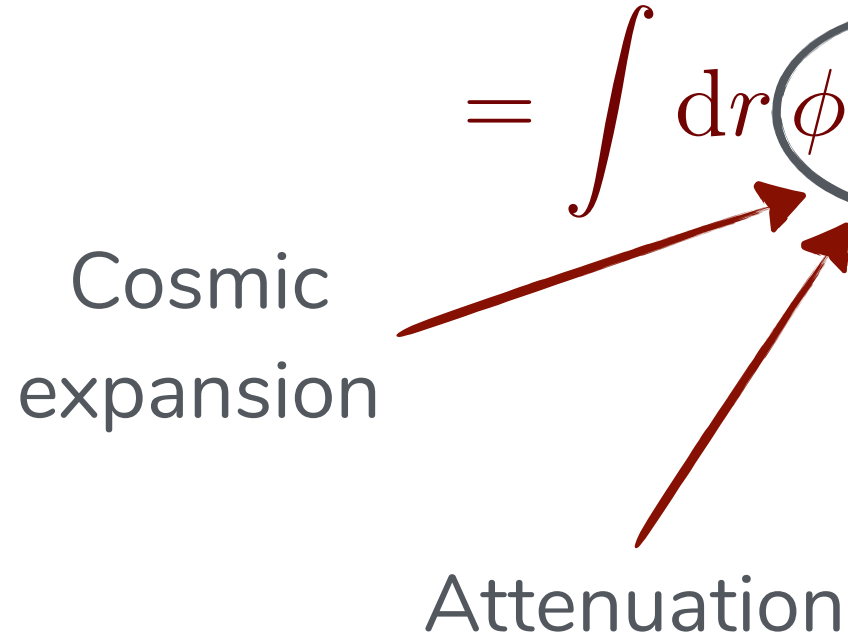
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Cosmic expansion

Attenuation



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Perturbations in the 3D number density of UHECR sources





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- Fluctuations in observed galaxy positions in a given direction on the sky and in a volume centred in a certain mean redshift

$$\Delta_{\text{g}}(\hat{\mathbf{r}}, z) = \frac{\text{NumberDensity}(\hat{\mathbf{r}}, z)}{\text{MeanNumberDensity}(z)} - 1$$

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Radial selection function





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3D galaxy  
density field

# Harmonic-space power spectrum

- Assuming that all UHECRs originate from observed galaxies

$$C_\ell = \int dr \frac{\phi_a(r) \phi_b(r)}{r^2} P_{ab} \left[ z(r), k = \frac{\ell + 1/2}{r} \right]$$

$$a, b = \{\text{CR}, \text{g}\}$$

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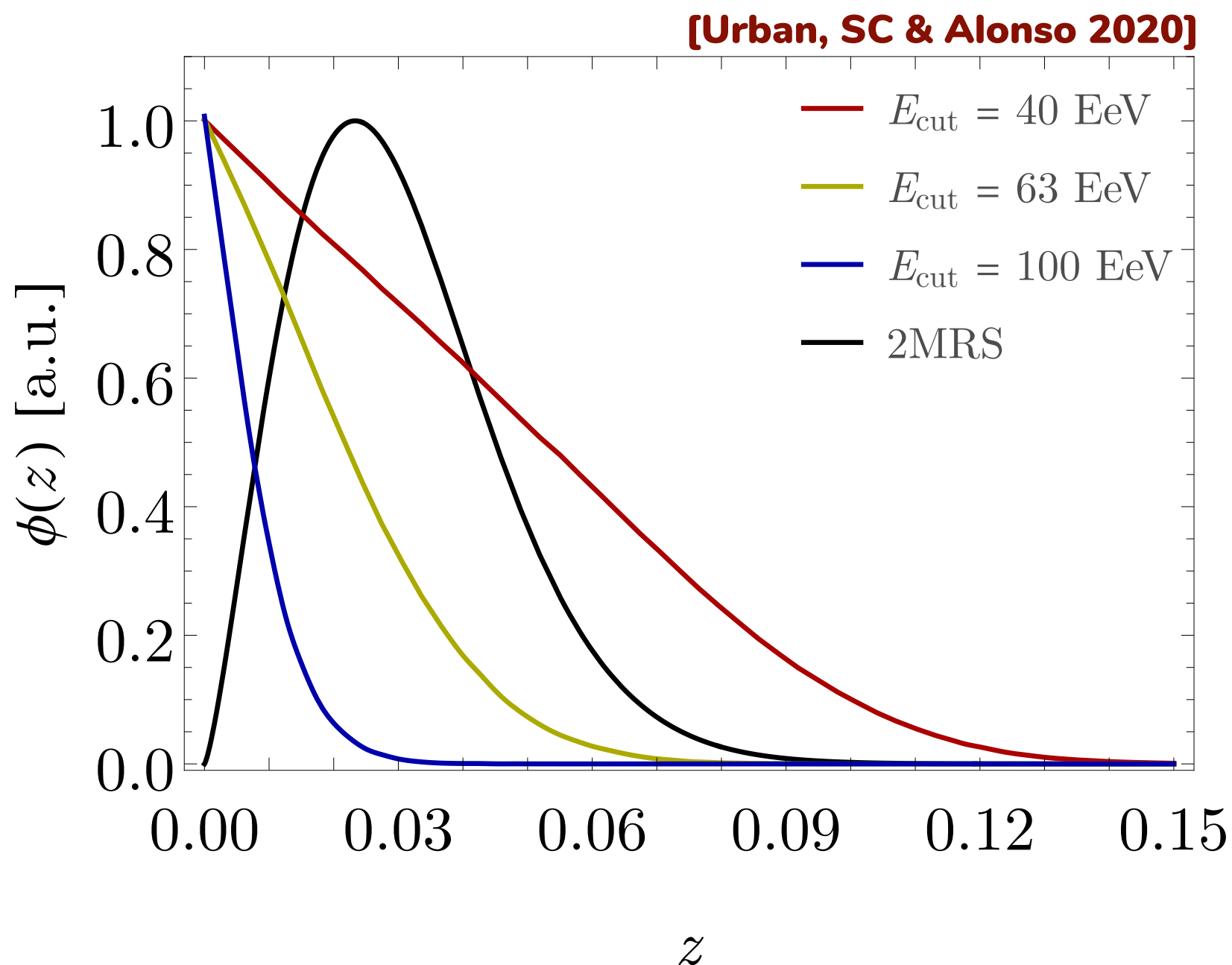
Fourier-space power spectrum  
of the two fields (**CR** and/or **g**)



# How does it look like?



- UHECR and 2MRS (2MASS spectro-z survey) galaxy kernels

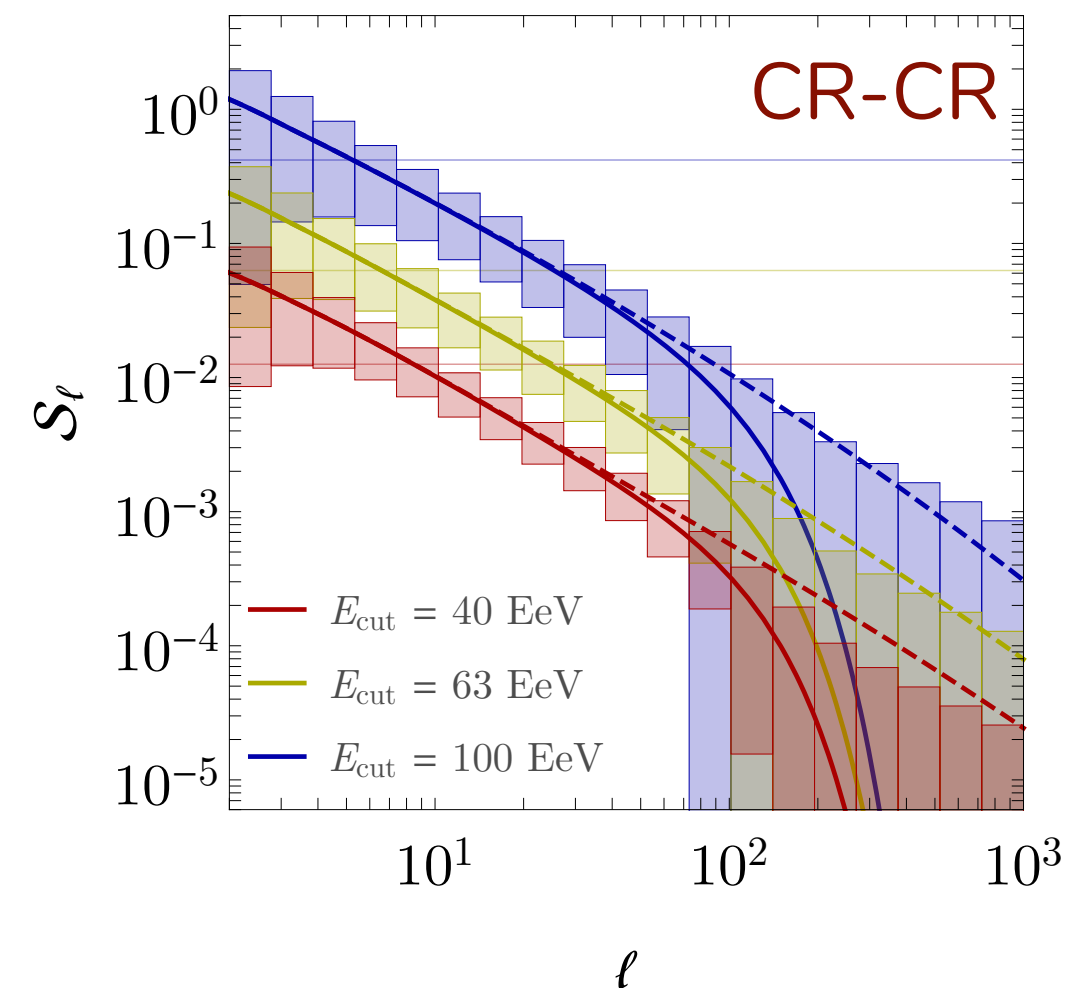


# How does it look like?



- Auto-correlation of UHECRs

[Urban, SC & Alonso 2020]

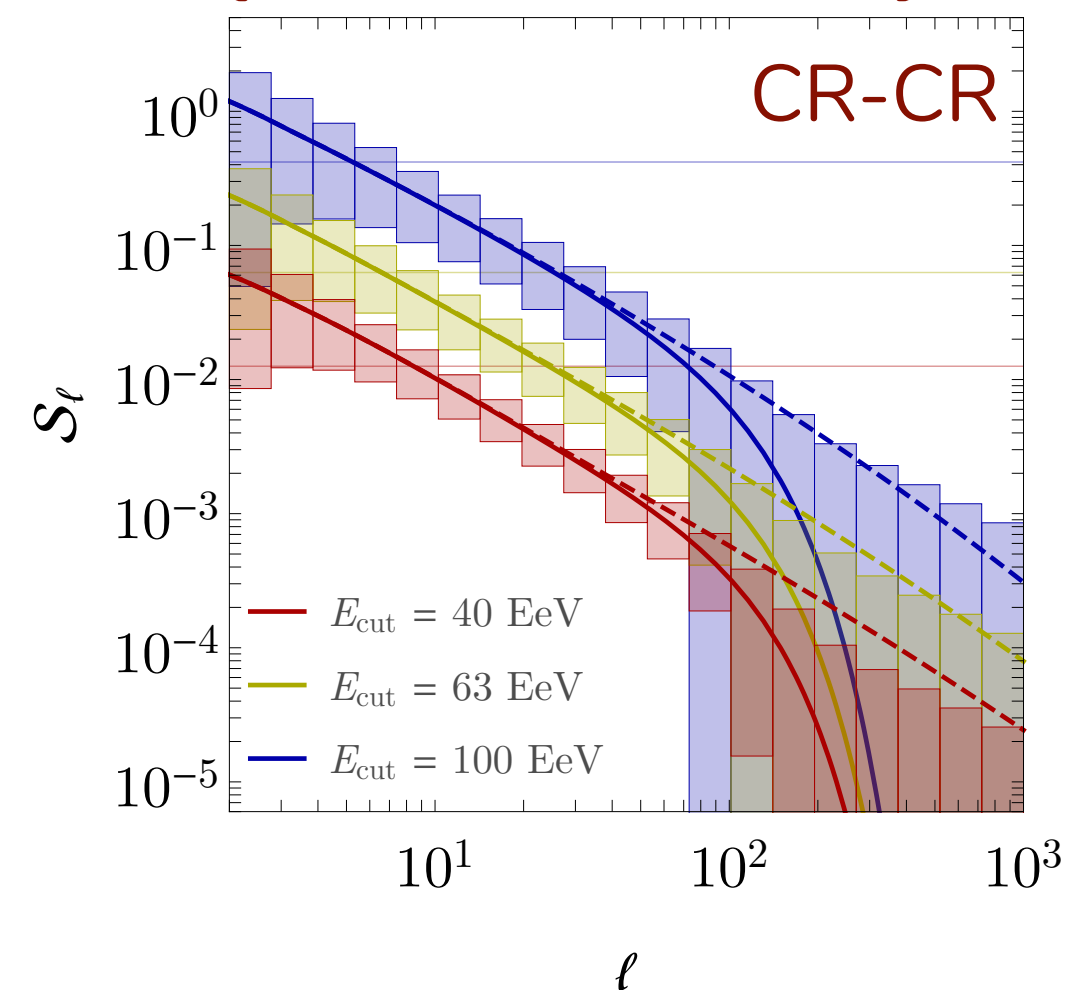


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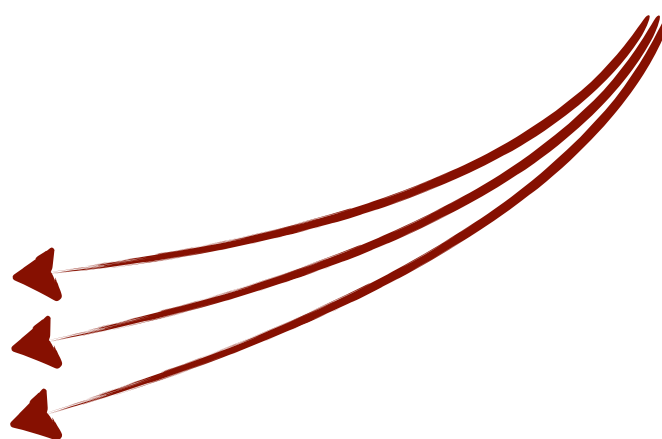


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[Urban, SC & Alonso 2020]



Not including a  $1^\circ$  beam  
smearing accounting  
for (x)GMF



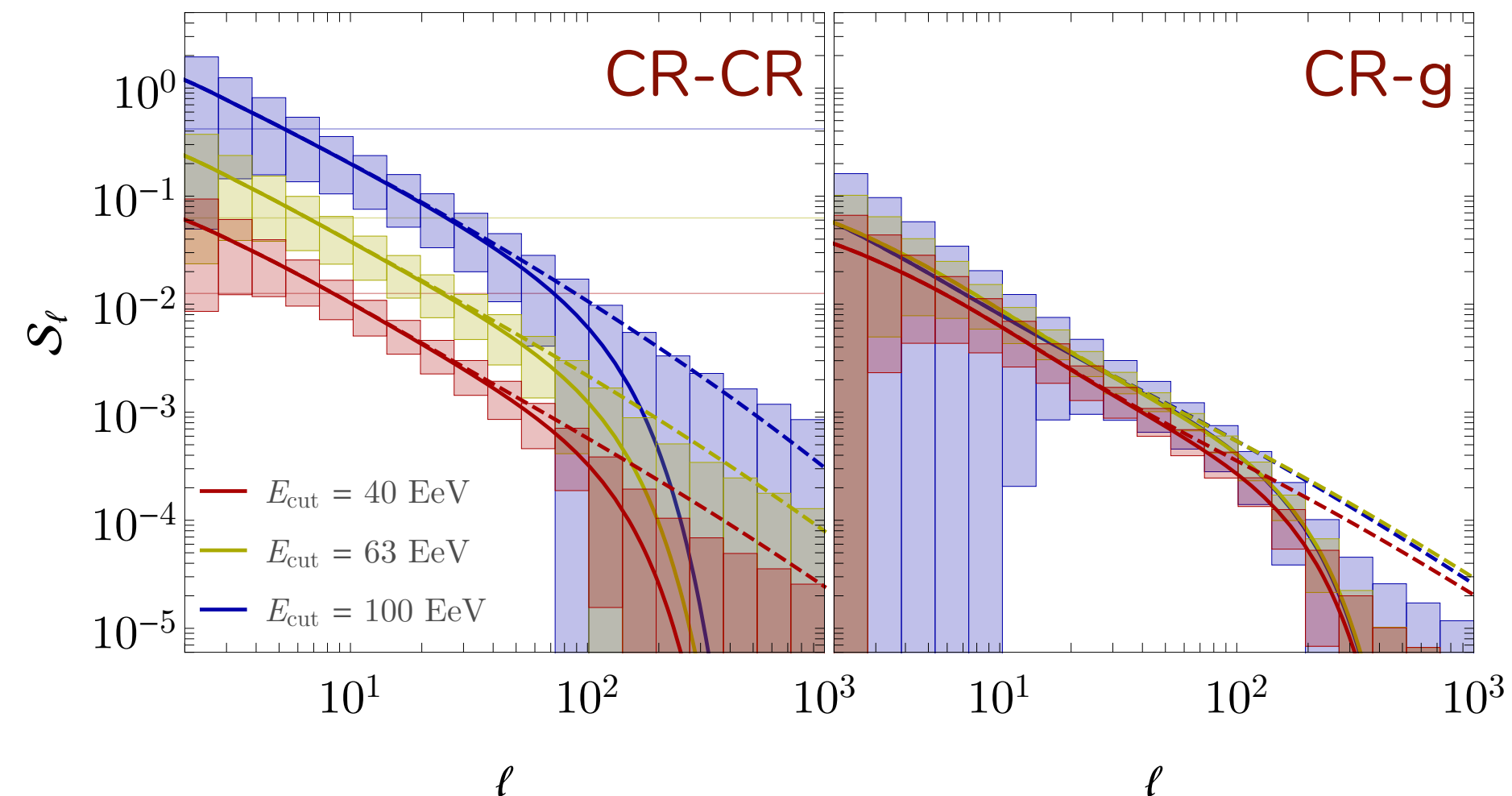


# How does it look like?



- Cross-correlation between UHECRs and 2MRS galaxies

[Urban, SC & Alonso 2020]

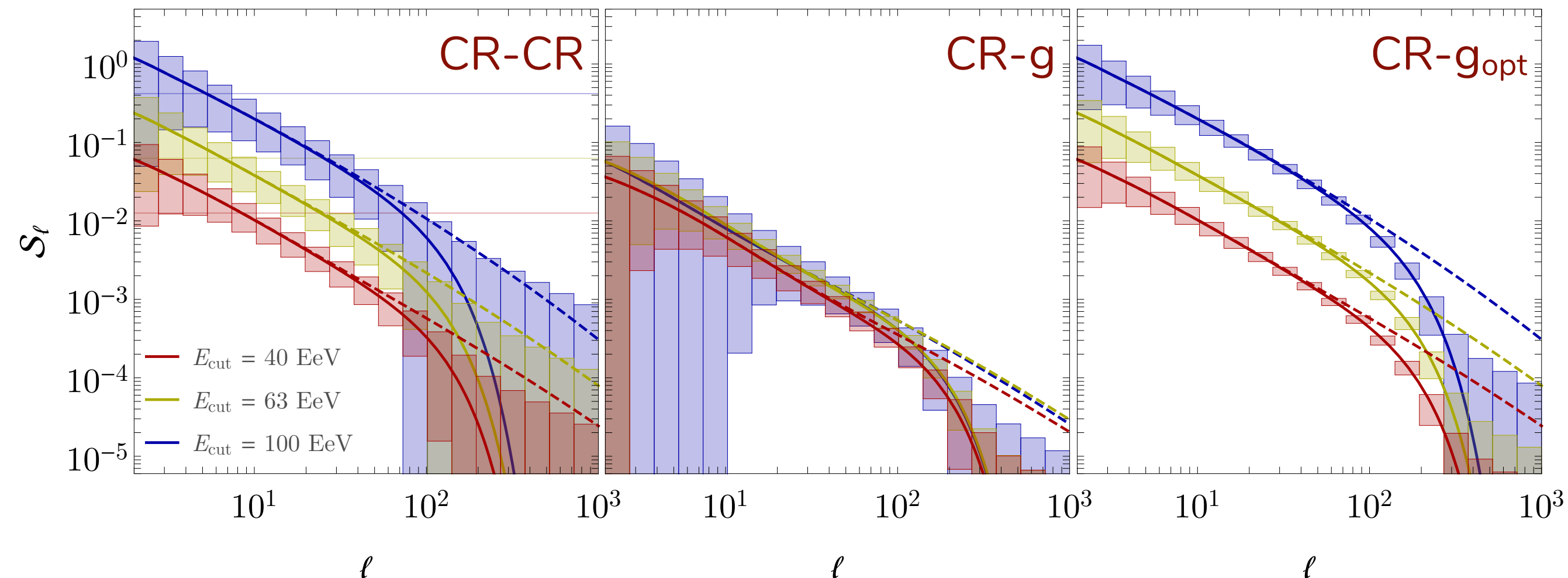


# How does it look like?



- Cross-correlation between UHECRs and 2MRS optimally-weighted galaxies

[Urban, SC & Alonso 2020]



# Signal-to-noise ratio



- Per-multipole signal-to-noise ratio

$$\text{SNR}^2 = \sum_{l=l_{\min}}^{l_{\max}} (\text{SNR}_l)^2$$

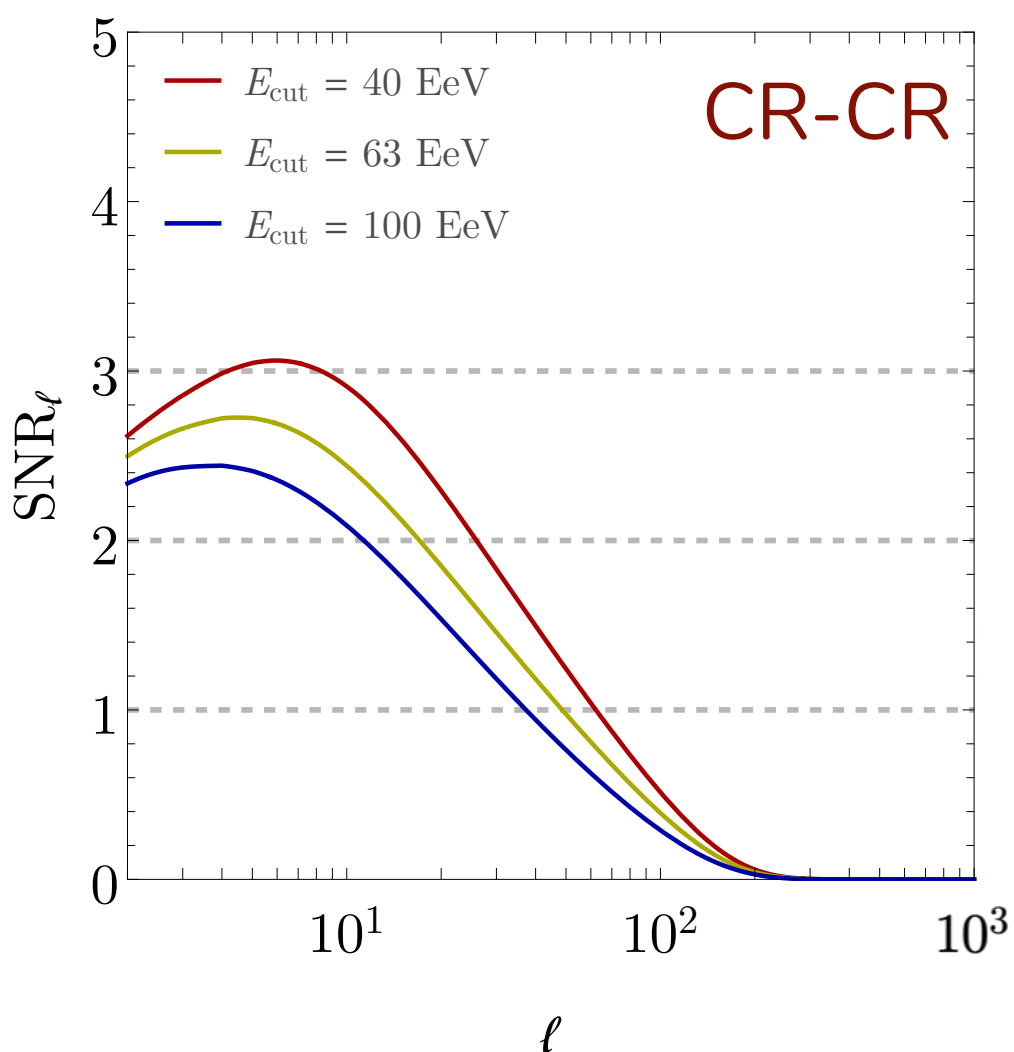
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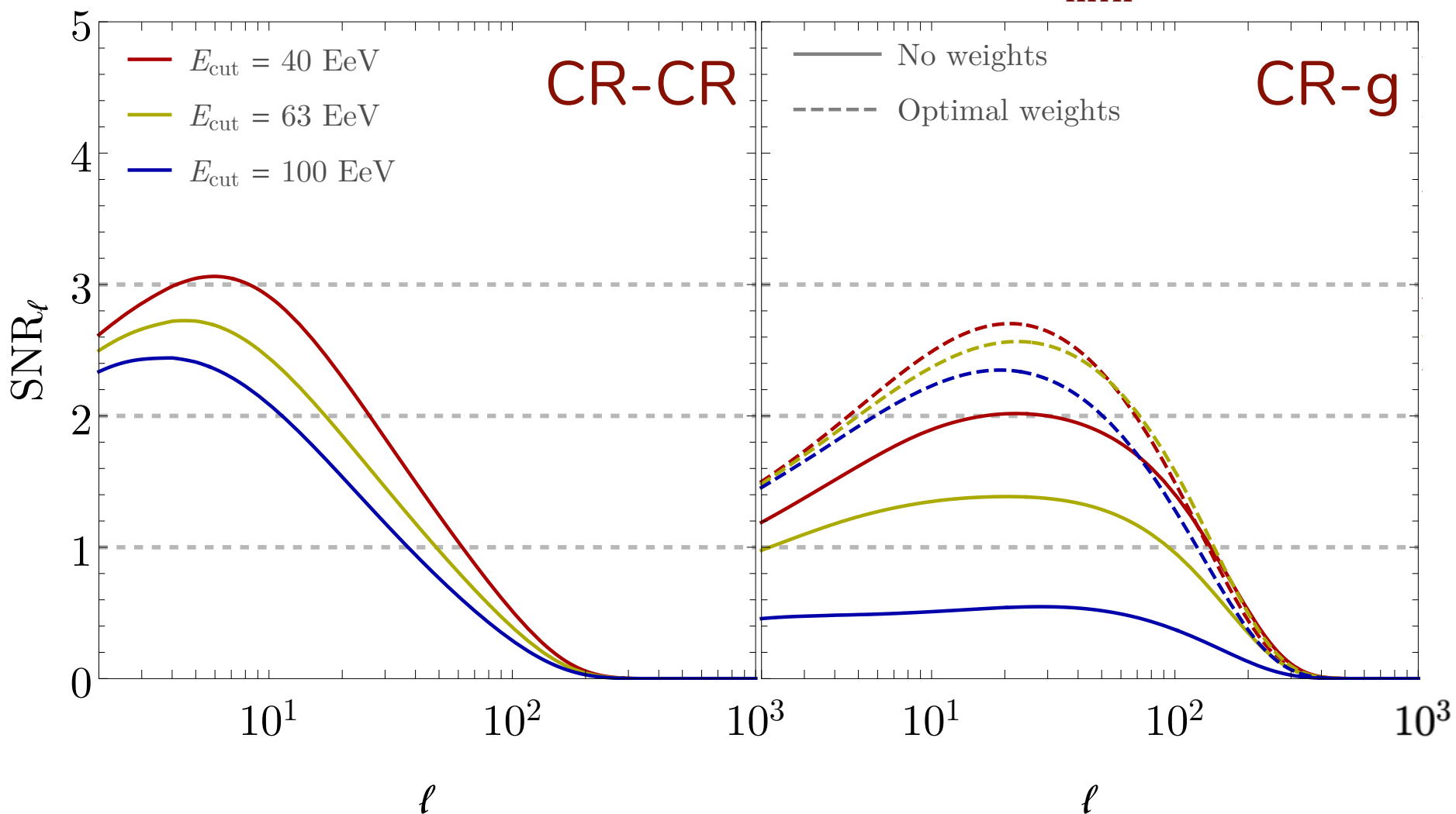


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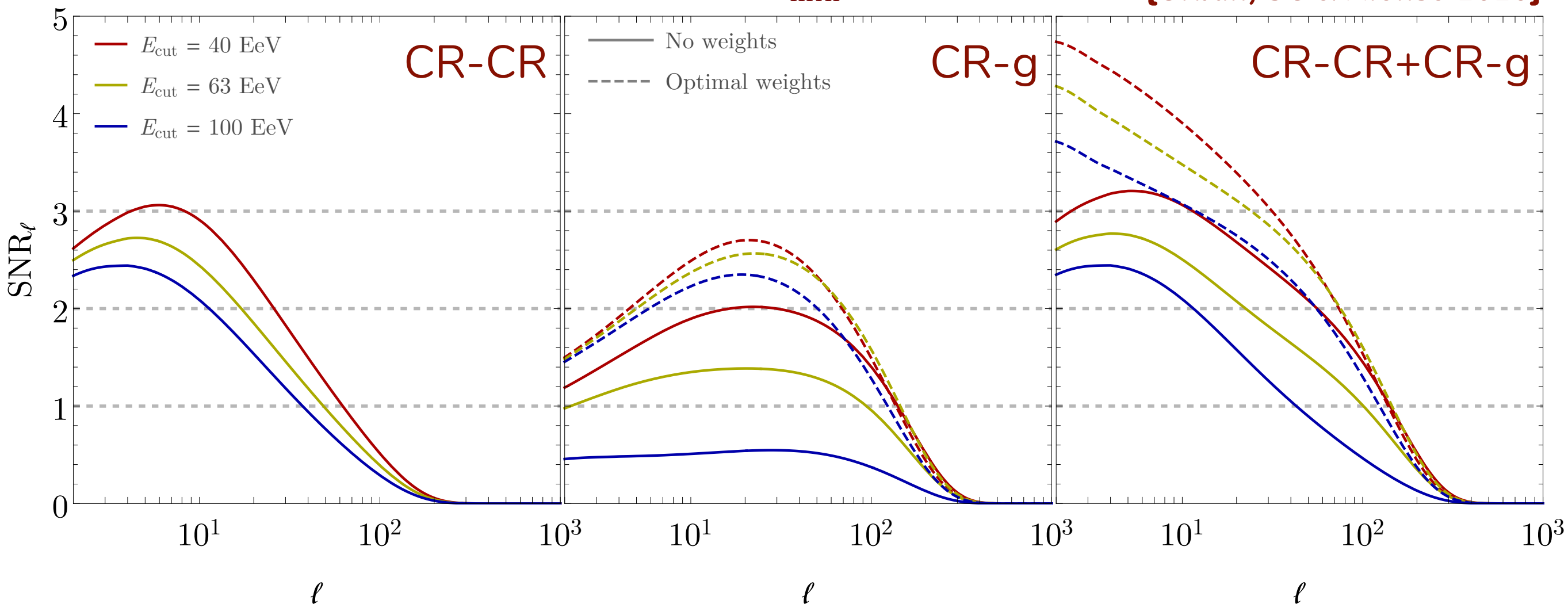


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# Conclusions

- A new observable for UHECR physics:
  - The harmonic-space cross-correlation power spectrum between the arrival directions of UHECRs and the distribution of the cosmic large-scale structure as mapped by galaxies
- Easier to detect than UHECR alone (galaxies boost the signal)
- More sensitive to anisotropies on small angular scales
- Less prone to systematic effects and to the GMF smearing