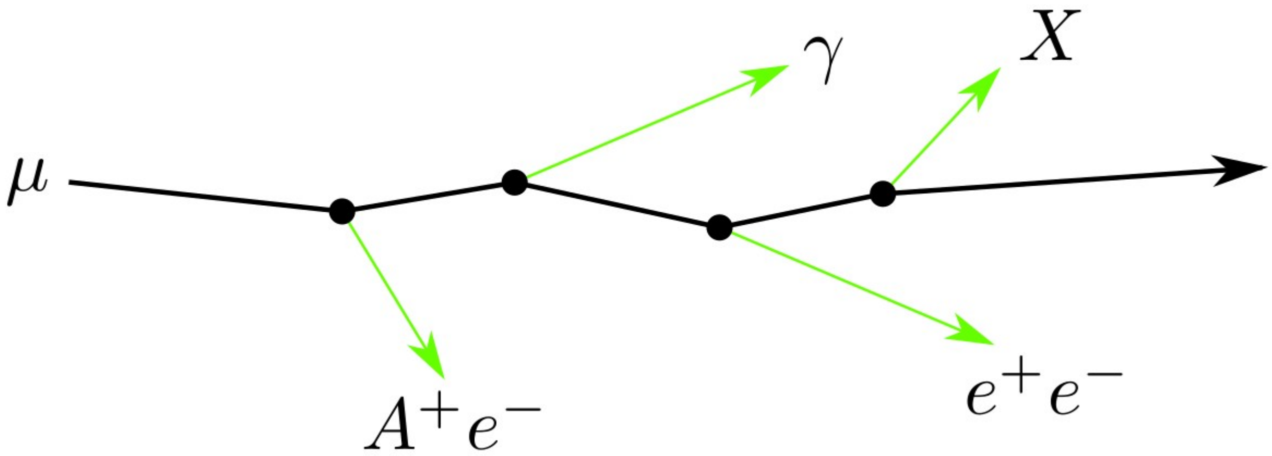

Uncertainties of the energy loss by inelastic interactions of muons with nuclei

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International Cosmic Ray Conference

Berlin/Online

2021

Muon energy loss processes

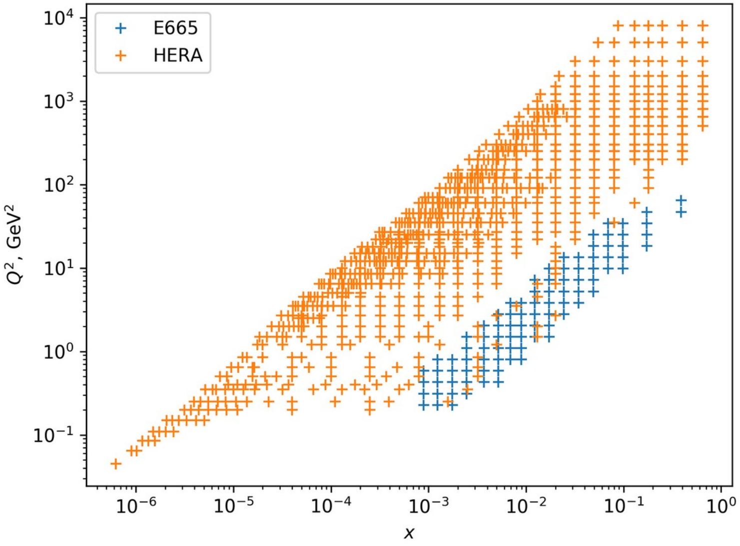
- Muons loose energy by four processes
  - Ionization
  - Electron-positron pair production
  - Bremsstrahlung
  - Nuclear interaction
- Largest uncertainties in description of nuclear interaction
  - Mainly small momentum transfer $Q^2 \rightarrow$ perturbative QCD is not directly applicable
  - Phenomenological parametrizations have to be used, which contain free parameters fitted to data

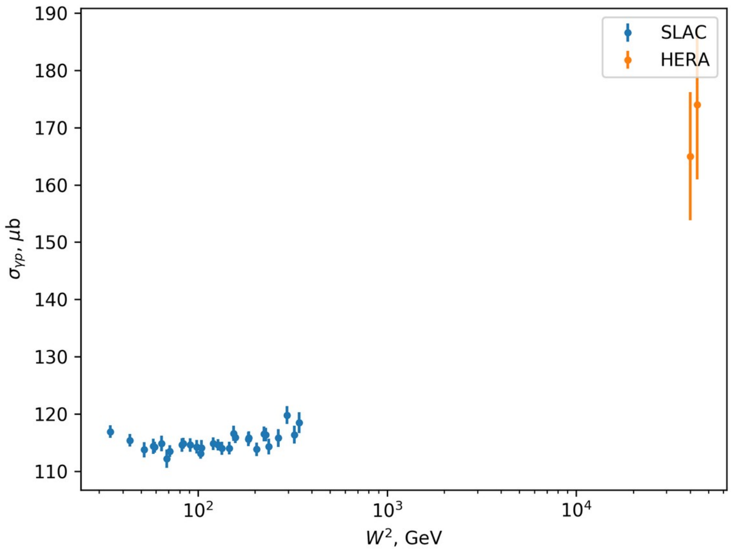
Description of nucleon structure functions in the low-x, low-Q^2 region

Approaches used in muon transport codes

- Regge theory-inspired parametrizations (Abramowicz et al. 1991, Abramowicz & Levy 1997; Block et al. 2014)
  - Based on the analyticity of amplitudes as functions of complex variables
- Vector meson dominance (Bezrukov & Bugaev 1980, 1981; Bugaev & Shlepin 2003)
  - Description of photohadronic interactions via intermediate vector mesons (ρ, ω, φ and heavier excitations ρ', ρ'' etc.)
- Disadvantage of many parametrizations: neglect of limiting kinematic regions
- First attempt to cover the whole kinematic region: Petrukhin & Timashkov 2004
- At high energies, we have to extrapolate beyond the region covered by experimental data

Datasets used in this study





Uncertainty propagation

- Average energy loss is a double integral over inelasticity y and momentum transfer $Q^2 \rightarrow$ cannot in general be taken analytically
- Calculate gradient of structure functions with respect to fit parameters by automatic differentiation $\rightarrow$ obtain via Leibniz‘ integral rule gradient of energy loss with respect to fit parameters
- Obtain covariance matrix Σ from fit

$$\sigma_f(X_1, \ldots, X_n) = \sqrt{\sum_{ik} \frac{\partial f}{\partial X_i} \Sigma_{ik} \frac{\partial f}{\partial X_k}}$$

ALLM parametrization

- 23 free parameters
- Developed in Abramowicz et al. (1991) and fit to fixed-target data
- Later fit repeated with early HERA data (Abramowicz & Levy 1997)
- Recently repeated in Abt et al. (2017) with combined HERA data
  - Best-fit mathematically ill-defined in photoabsorption limit $Q^2 \to 0$
- Best-fit on the data used χ^2/ndf = 1.01
- Refit similar to ALLM 97 parametrization

Bezrukov & Bugaev and Bugaev & Shlepin

- Developed in Bezrukov & Bugaev (1980, 1981) on the basis of the generalized vector meson dominance model
- Numerical calculations there were carried out using a large number of intermediate mesons
- Commonly used approximation with two effective masses was developed as a useful approximation with an accuracy of about 5%, the typically used closed analytic formula was achieved by approximate analytical integration
- In Bugaev & Shlepin (2003) the hard component was calculated and parametrized based on the color-dipole model of Forshaw, Kerley & Shaw (1999)
- Best-fit of the commonly-used approximation has on these data best-fit $\chi^2/\mathrm{ndf} \sim 6$
- $\langle -dE/dX \rangle$ calculated from refit rises slower with increasing energy than original work

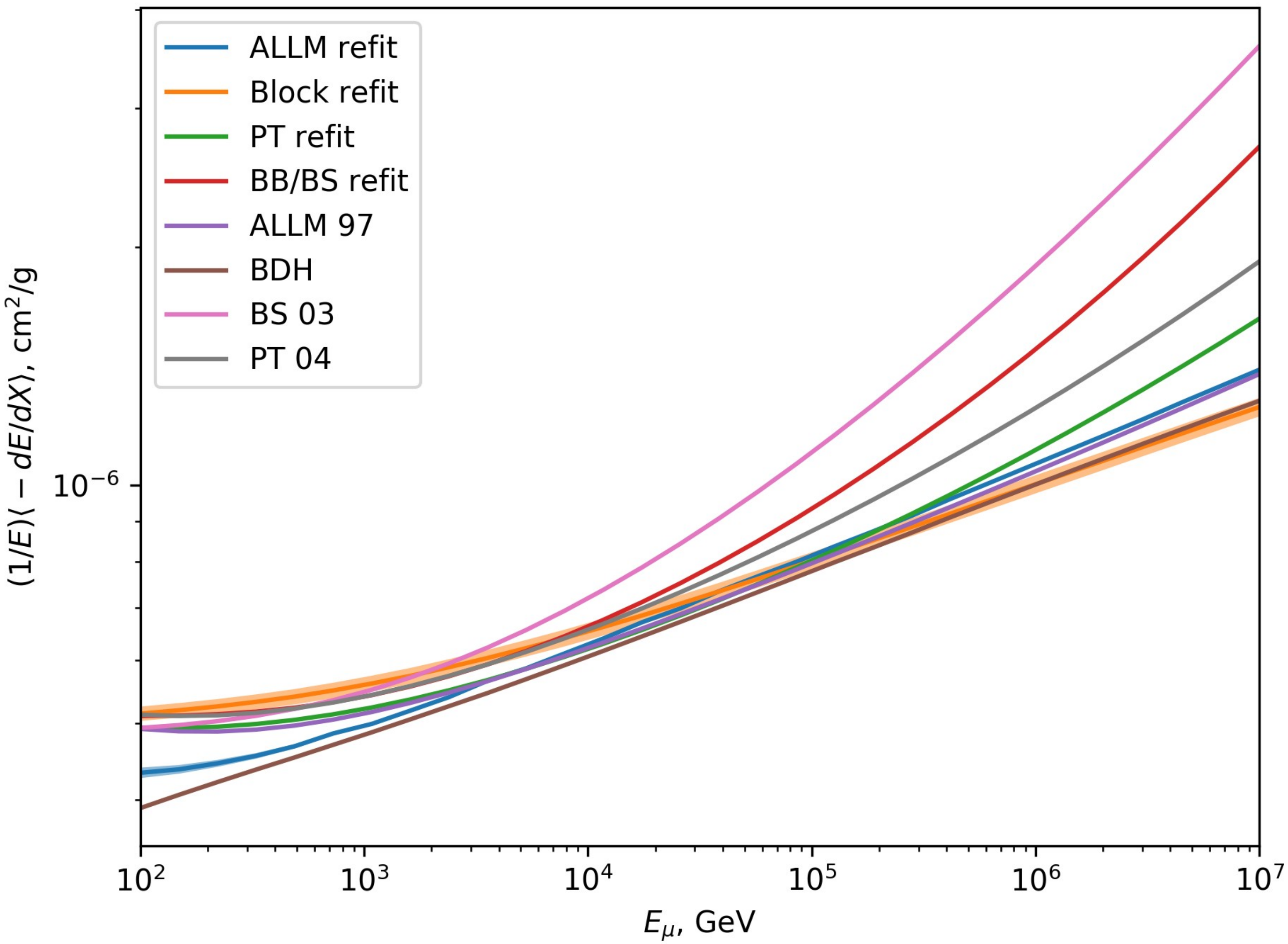
Petrukhin & Timashkov

- Developed on the basis of vector meson dominance, Regge theory and leading order DGLAP and BFKL equations, taking into account the limiting kinematical regions of photoproduction, quasielastic scattering and deep-inelastic scattering
- Able to explain the old data within 10–15%
- Small number of parameters
- Best-fit on the data used χ^2/ndf ~ 8
- $\langle -dE/dX \rangle$ calculated from refit similar to ALLM results up to energies of the order of 100 TeV

BDH parametrization

- Developed in Block, Durand & Ha (2014), assuming a saturated Froissart-bound, i.e. $\sigma \propto \ln^2 W$
- 12 free parameters, of which 3 where fixed by requiring that the photoabsorption cross section coincides with the fit of Block & Halzen (2004)
- We repeat their fit on E665 and combined HERA data, using as photoabsorption limit the HPR_1R_2 parametrization by Belousov et al. (2016)
- Uncertainty of HPR_1R_2 is smaller, because – assuming hadron universality – the fit includes other hadronic cross sections such as *pp, Kp, πp*, …
- Best-fit on data used $\chi^2/ndf = 1.10$

Muon energy loss



Conclusion

- We reconsidered several commonly-used structure function parametrizations in the light of the precise combined *ep* scattering data from HERA experiments H1 and ZEUS and other DIS data at low x and low Q^2
- Refit ALLM and BDH energy loss predictions agree within 5% around 100 TeV, the predictions slowly diverge from each other at higher energies, reaching 10% at energies of several PeV
- Refit of ALLM parametrization has best χ^2/ndf, but also the by far largest number of parameters; physical significance of all parameters difficult to ascertain
- Petrukhin-Timashkov parametrization has smallest number of parameters, but performs less well on newer experimental data
- All parametrizations agree within 10–15% at lower energies, serious disagreement at high energies
- Further work necessary from experimental and theoretical side
  - New models, describing the existing data with clear physical foundation
  - New data, in particular at higher energies