Time-dependent treatment of cosmic-ray spectral steepening due to turbulence driving

Martin Pohl

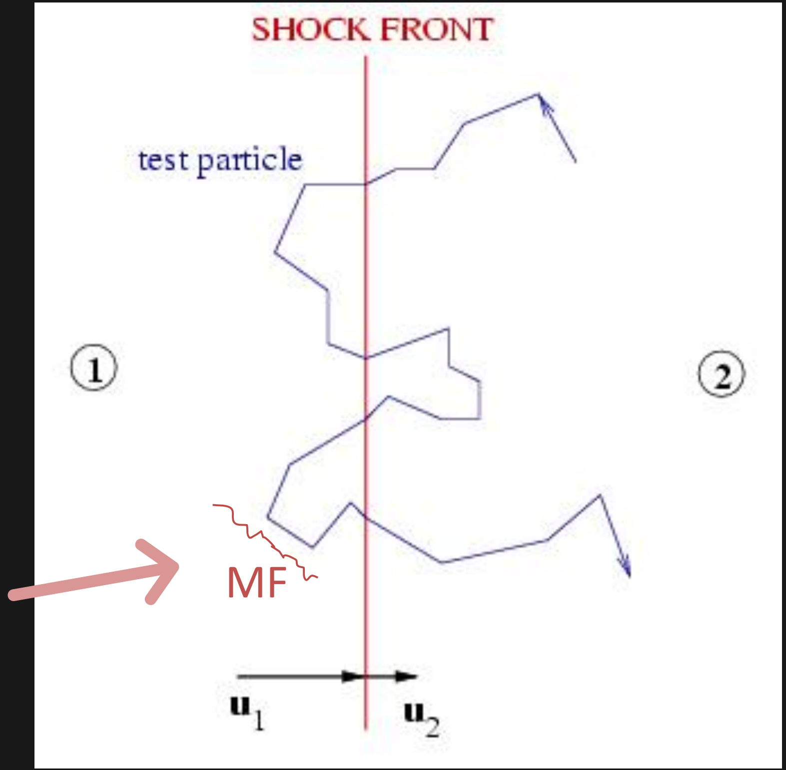
Acceleration process

Shock acceleration:

Significant energy gain
by multiple shock crossing

Test-particle spectrum E^{-2}

Requires continuous driving of turbulence



Question

Turbulence driving implies energy loss

Is there an impact on CR spectra?

Time dependence

Nonresonant modes (aka Bell)

Early analysis in steady-state limit:

Spectral steepening can be significant

Is there enough time to reach the steady state?

Is there additional energy transfer to heat, etc?

1st method

Peak growth rate

$$\gamma_{\max} \simeq \Omega_{\rm p} \, \frac{v_{\rm sh}}{2\, v_{\rm A}} \frac{N_{\rm cr}}{N_{\rm p}} = \omega_{\rm p,p} \, \frac{v_{\rm sh}}{2\, c} \frac{N_{\rm cr}}{N_{\rm p}}$$

Energy density transfer

$$\dot{U} \simeq \int dk \, \gamma(k) \, \frac{B_k^2}{4\pi\epsilon} \lesssim \gamma_{\max} \, \frac{(\delta B)^2}{4\pi\epsilon}$$

Insert and take ratio

$$\tau_{\rm loss} \simeq \frac{U_{\rm cr}}{\dot{U}} \gtrsim \frac{2\epsilon\Gamma_{\rm cr}}{\omega_{\rm p,p}} \frac{U_{\rm bulk}}{U_{\delta B}} \frac{c^3}{v_{\rm sh}^3}$$

1st method

Comparison with acceleration time requires diffusion coefficient

$$\kappa = \eta r_{\mathrm{L}} c/3$$

Yields spectral modification

$$\Delta s \lesssim \frac{2\,(s-1)\,\eta\,M_{\mathrm{A}}}{3\,\epsilon}\,\frac{U_{\delta B}}{U_{\mathrm{bulk}}}$$

Here M_A is written with the full field amplitude

2nd method

Integrate energy loss rate over the entire precursor

$$\dot{E}_{\rm tot} \lesssim \frac{\omega_{\rm pp,sh}\, N_{\rm cr,sh}}{8\pi\,\epsilon\, c\, N_{\rm p,sh}\sqrt{v_{\rm sh}}} \int_{r_{\rm sh}}^{\infty} dr\, \frac{v(r)^{3/2}\,(\delta B(r))^2}{\exp\left(\int_{r_{\rm sh}}^{r} dr'\,\frac{v(r')}{\kappa(r')}\right)}$$

Allows treatment of spatial variations in precursor

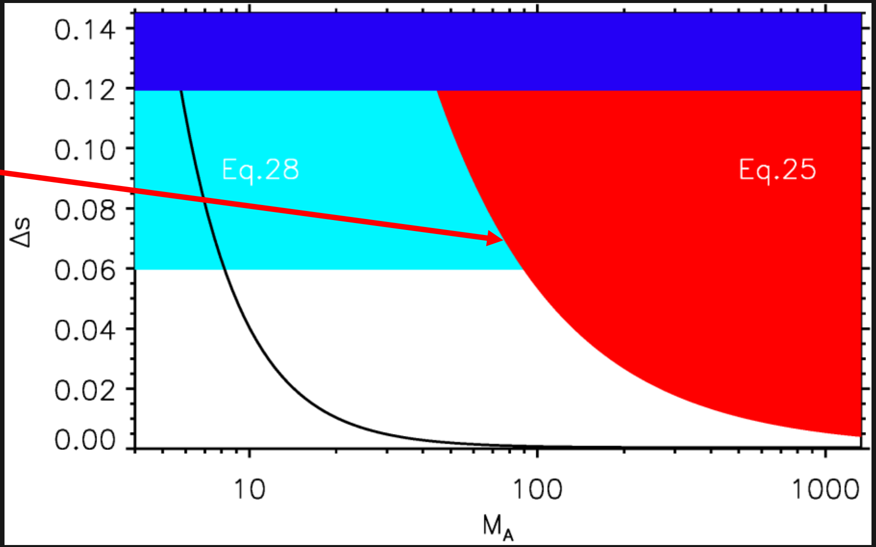
They don‘t matter → Steepening $\lesssim \frac{2\,(s-1)^2\,\eta\, M_{\rm A}}{3\,\epsilon}\,\frac{U_{\delta B}}{U_{\rm bulk}}$

Generalization

Replace Alfvenic Mach number for $\delta B >> B_0$

$$\Delta s \lesssim \frac{2\,(s-1)^2\,\eta}{3\,\epsilon\,M_A}$$

$$= \frac{2\,(s-1)^2\,\eta}{3\,\epsilon}\sqrt{\frac{U_{\delta B}}{U_{\rm bulk}}}$$

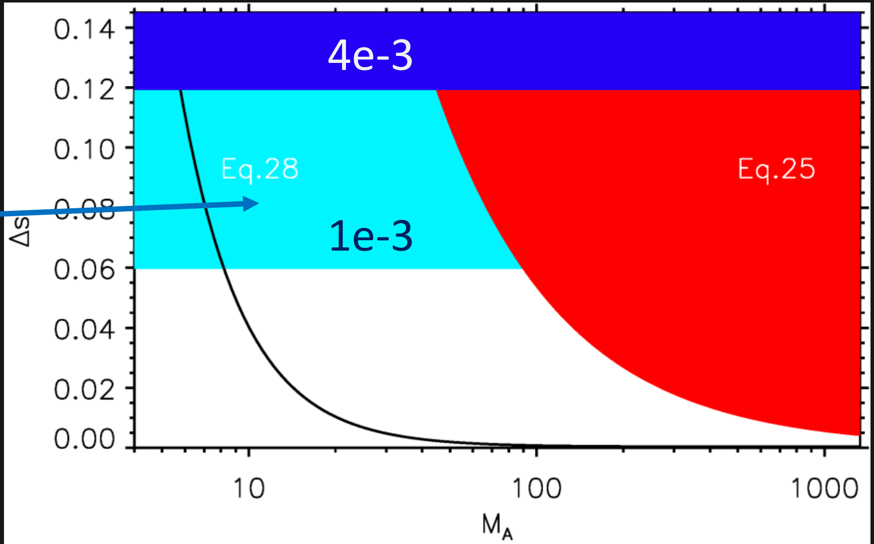


Generalization

Bell‘s instability requires $kr_L \gg 1$ or $\frac{v_{sh}}{4c}\frac{U_{cr}}{U_{\delta B}} \gg 1$

which gives

$$\Delta s \lesssim \frac{(s-1)\eta}{3\sqrt{2}\,\epsilon}\sqrt{\frac{v_{sh}}{c}\frac{U_{cr}}{U_{bulk}}}$$



These are upper limits!

Summary

Spectral steepening on account of turbulence driving happens

Its level is limited even for fast shocks, $\Delta s < 0.1$.

Important are the time and space available.

Earlier steady-state estimate gives less than one growth time